dicate fairly strong sidewall interferences for aspect ratios below $\Lambda = 2$, in contrast to most of the theories. The curve labeled "Menard" is the exception, but this result is questionable since a physically unrealistic flow model is being used and slight changes in the assumptions yield quite different results (smaller corrections).[4]

The reason for the inadequacy of the theoretical methods is seen in the insufficient consideration of three-dimensional effects. Some indication of these effects is given in Fig. 4. It shows pronounced pressure variation across the span as measured by a two-spot laser velocimeter traversing four chordwise positions.[8] The traverse stations are indicated, together with a sketch of the airfoil.

These findings confirm that the basic assumption made in all theoretical approaches, that a global correction can be used to account for sidewall interference effects, must be questioned. The effects are due, in large measure, to the three-dimensional character of the flow originating from the mutual interaction between the sidewall boundary layers and the pressure field produced by the airfoil. Simple global corrections to the mainstream conditions do not adequately account for such effects.

References

[1] Ganzer, U., "On the Use of Adaptive Walls for Transonic Wind Tunnel Testing," AGARD-CP-335, Sept. 1982, Paper 13.

[2] Rainbird, W. J., "Design and Use of Boundary Layer Control for 2D Aerofoil Testing: 15 in. × 60 in. Section," National Aeronautical Establishment, Ottawa, Memo. HSA-40, Feb. 1970.

[3] Chan, Y. Y., "Wall Boundary Layer Effects in Transonic Wind Tunnels," AGARD-CP-335, Sept. 1982, Paper 7.

[4] Chevallier, J.-P., "Effects tridimensionnels sur les profils," ONERA TP 1981-117, Nov. 1981.

[5] Sewall, W. G., "Effects of Sidewall Boundary Layers in Two-Dimensional Subsonic and Transonic Wind Tunnels," *AIAA Journal*, Vol. 20, Sept. 1982, pp. 1253-1256.

[6] Ganzer, U. and Ziemann, J., "Profilmessungen bei verschiedenen Seitenwandabständen im 1 × 1 m Transsonik-Kanal der DFVLR," TU-Berlin ILR-Mitteilung 102, 1982.

[7] Elsenaar, A. and Stanewsky, E., "A Report of a GARTEur Action Group on Two-Dimensional Transonic Testing," AGARD-CP-335, Sept. 1982, Paper 5.

[8] Hähnel, B. and Kuczka, D., "Messungen drei-dimensionaler Effekte bei transsonischen Profilströmungen mittels eines Laser 2 Fokus-Velocimeters," TU-Berlin ILR-Mitteilung 106, 1982.

Frozen Plasma Boundary-Layer Flows over Isothermal Flat Plates—Parametric Study

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Introduction

THE nonlinear partial-differential equations describing most boundary-layer problems are difficult to solve. Consequently, many investigators resorted to using simplifying similarity transformations. In complex flows, where similarity solutions cannot be used, an exact solution for the general boundary-layer flow equations is not possible. Reviews of commonly used techniques for solution of boundary-layer problems can be found in Refs. 1-3.

One type of boundary-layer problem is the shock-wave-generated boundary layer. For strong shock waves the shock-induced flow must be considered as a real gas, i.e., plasma. The ionized gas can be in either a nonequilibrium, equilibrium, or frozen state, depending on the time given to the flow to relax to its appropriate equilibrium state.

In a recent work of Liu et al.,[4] the shock-tube-generated flat-plate laminar boundary layer in argon was studied. In their study the governing equations were solved using an implicit six-point finite difference scheme for the three cases, i.e., nonequilibrium, equilibrium, and frozen flow. It is of interest to note that the numerical results obtained for the frozen flow case lie closer to the experimentally measured flow properties than the nonequilibrium values (see Ref. 4, Figs. 13-15 and 18-20). Consequently, it is apparent that the frozen behavior could represent quite well the shock-tube-generated flowfield over a flat plate and should not be considered only as a purely academic exercise.

Motivated by this fact and the known difficulties encountered in using the finite difference method for ionizing boundary-layer flows, we propose a simple numerical approach to solve the frozen case of a shock-tube-generated boundary-layer flow. This approach was successfully used for a simple case of perfect gas flowing over the flat plate.[5] The present approach is based on the following "group property" of Eqs. (1-3): The solution of Eqs. (1-3), i.e., $\phi(\beta)$, $H(\beta)$, and $\alpha(\beta)$, is invariant under the transformation $\phi(\beta) = b\phi_0(a\beta)$, $H(\beta) = cH_0(a\beta)$, and $\alpha(\beta) = d\alpha_0(a\beta)$. The idea for using this type of transformation is due to Topfer,[6] who solved Blasius' equation using the transformation $\phi(\eta) = \alpha\phi_0(\alpha\eta)$. Details concerning our solution can be found in Ref. 7.

Theoretical Background

For a supersonic flow over a flat plate one can use the following assumptions: 1) two-dimensional steady flow; 2) laminar flow; 3) no (continuum) radiation losses; 4) no diffusion ($u = u_e$); 5) constant freestream conditions along the flow direction; 6) no electric or magnetic fields; 7) thermal equilibrium ($T = T_e$); 8) frozen flow, i.e., $\dot{\omega}_e = 0$; 9) constant Le number; 10) constant Pr number; 11) $\rho\mu = H^n$ [this type of relation is based on the fact that for ideal gases $\rho\mu = (P/RT)C_1 T^{0.76} = C_2 T^{-0.24}$; hence for nonideal gases a similar relation is assumed with an unknown power $-n$]; and 12) $I \gg 5/2RT$ (i.e., $T \ll 2/5\theta_I$, where T is the plasma temperature and θ_I is the characteristic ionization temperature, for argon $\theta_I = 182{,}850$ K). The following set of self-similar equations then can be obtained.[7]

$$\phi\phi'' + (H^n\phi'')' = 0 \tag{1}$$

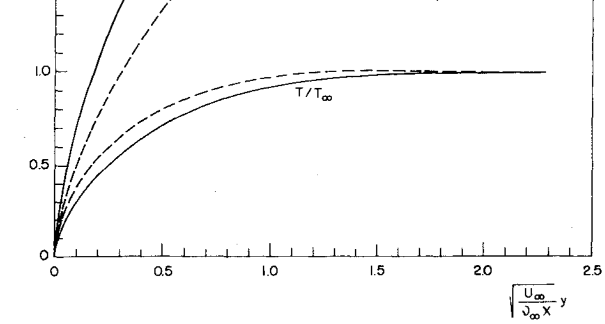

Fig. 1 Velocity and temperature dependence on *Pr*.

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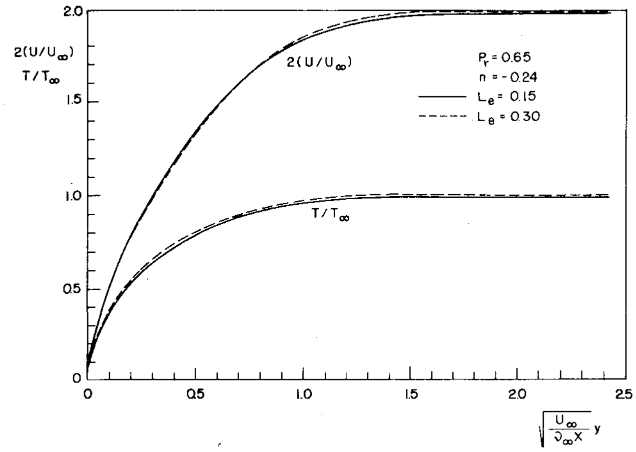

Fig. 2 Velocity and temperature dependence on *Le*.

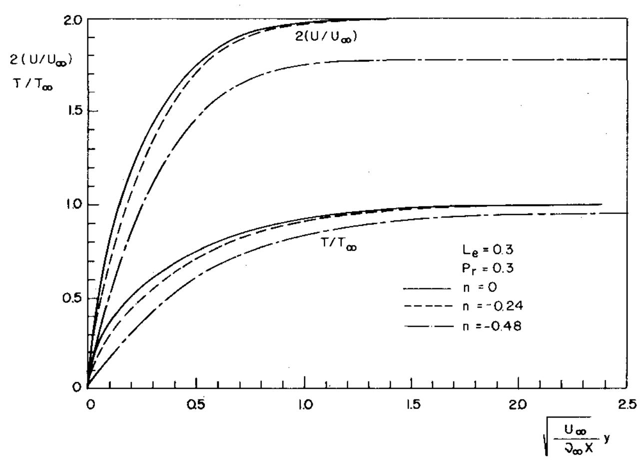

Fig. 3 Velocity and temperature dependence on *n*.

$$Pr\phi H' + (H^n H')' + (A/4)(H^n \phi' \phi'')' + BC(H^n \alpha')' = 0 \quad (2)$$

$$Pr\phi\alpha' + Le(H^n\alpha')' = 0 \quad (3)$$

where ϕ is the velocity potential function, H the normalized total enthalpy, and α the degree of ionization. Pr and Le are the Prandtl and Lewis numbers. A, B, and C are constants depending on the freestream flow properties, and n is the exponential dependence of the density-viscosity product on the temperature as defined in assumption 11.

In the case of an isothermal plate the boundary conditions are: $\phi(0)=0$, $\phi'(0)=0$, $H(0)=H_W$, $\alpha(0)=\alpha_W$, $\phi'(\infty)=2$, $H(\infty)=1$, and $\alpha(\infty)=1$. If the plate can be considered as relatively cold, then $\alpha_W=0$.

Since not all the boundary conditions are given at $\beta=0$ (i.e., some are given at $\beta=\infty$) the problem at hand is known as a two-point boundary-value problem. One method for solving two-point boundary-value problems is known as the shooting method.

The present results are based on a new approach for solving Eqs. (1-3). In this approach a transformation by which the problem is reduced to a Cauchy problem was found (for details, see Ref. 7).

Results and Discussion

Equations (1-3) enable us to solve the laminar boundary layer that develops over a flat plate when a compressible, singly ionized gas flows over it. Such flows can be induced by strong shock waves. For this case, the postshock equilibrium flow properties, which are obtained behind the relaxation zone, are the freestream properties for the boundary-layer flow.

In our analysis we assumed that the Prandtl and Lewis numbers are constants. In general they are functions of the plasma temperature and its degree of ionization. When the plasma is in a state of thermal equilibrium its degree of ionization can be expressed in terms of the temperature and pressure through the Saha equation, and since for boundary-layer problems with constant freestream pressure the pressure throughout the entire field can be considered constant, it appears that for the problem at hand Pr and Le depend on the temperature only. (See Ref. 3, Fig. 13.)

Liu[3] studied the laminar boundary-layer problem for nonequilibrium, equilibrium, and frozen flow cases. In his solution of the frozen case he used $\rho\mu=1$ (i.e., $n=0$), $Pr=1$, and $Le=1$. These are unrealistic assumptions as can be verified from Fig. 13 of Ref. 3. In addition, setting $\rho\mu=1$, $Pr=1$, and $Le=1$, Eqs. (1-3) reduce to the following set:

$$\phi\phi'' + \phi''' = 0 \qquad \phi H' + H'' = 0 \qquad \phi\alpha' + \alpha'' = 0$$

Consequently, by using these unrealistic values, the coupling between the equations is removed. Now the first equation which is reduced to the well-known Blasius equation can be solved separately! Then each of the following two equations becomes independent, and, hence, can also be solved separately. Thus, by selecting these numerical values ($\rho\mu=1$, $Pr=1$, $Le=1$), Liu has artificially removed the inherent complexity associated with Eqs. (1-3), i.e., the coupling between these equations.

For an ideal gas, $n=-0.24$, in our solution we have used the following values: $n=0$ (identical to Liu[3]), $n=-0.24$ (the value appropriate for an ideal gas), and $n=-0.48$. Using this wide range of n enabled us to study the dependence of the solution upon its value. Similarly, we have chosen two different values for Le ($=0.3$ and 0.15) and Pr ($=0.3$ and 0.65). Thus, the dependence of the flowfield on Le and Pr was also studied. Our selected values for n, Le, and Pr insure that the coupling between Eqs. (1-3) remains throughout the entire solution.

The temperature and velocity dependence on Pr and Le is shown in Figs. 1 and 2, respectively. While a change from 0.3 to 0.65 in Pr indicates a noticeable change in the profiles (Fig. 1), a change from 0.15 to 0.3 in Le hardly affects the velocity and temperature profiles.

Figure 3 illustrates the dependence of the flow on n. It is clearly seen that the smaller n is (i.e., more negative), the longer it takes to reach the inviscid uniform flow properties (i.e., the boundary layer is thicker).

Conclusions

The boundary-layer equations of a partially ionized frozen flow over a flat plate has been solved using a new approach in which the problem at hand was reduced from a two-point boundary-value problem to a Cauchy problem, thus offering a simple, stable, and relatively inexpensive solution technique. The method was applied to a strong shock-induced argon flow over an isothermal flat plate.

Since the method requires constant values for the Prandtl and Lewis numbers and the exponential dependence of n of the density viscosity product $\rho\mu$ upon the temperature T, the dependence of the flow inside the boundary layer on Pr, Le, and n was investigated. It was found that while Pr and n strongly affect the obtained flowfield, the influence of Le is negligibly small.

As a closing remark, it is of interest to note that the findings of Liu et al.[4] indicated that the frozen behavior represents the shock-induced flowfield over a flat plate quite well and, hence, the frozen solution should not be considered as a purely academic exercise only.

References

[1]Schlichting, H., *Boundary Layer Theory*, McGraw-Hill, New York, 1960.

[2]Stewartson, K., *The Theory of Laminar Boundary Layers in Compressible Fluids*, Oxford Mathematical Monographs, Oxford University Press, Oxford, England, 1964.

[3]Liu, W. S., "The Analysis of Shock Structure and Nonequilibrium Boundary-Layer Induced by a Normal Shock Wave in an Ionized Argon Flow," University of Toronto, Canada, UTIAS Rept. 198, 1975.

[4]Liu, W. S., Whitten, B. T., and Glass, I. I., "Ionizing Argon Boundary Layer. Part 1. Quasi-Steady Flat-Plate Laminar Boundary-Layer Flows," *Journal of Fluid Mechanics*, Vol. 87, 1978, Pt. 4, pp. 609-640.

[5]Yakhot, A., Ben-Dor, G., Rakib, Z., and Igra, O., "A New Approach to the Solution of the Boundary Layer Equations of an Ideal Compressible Flow over a Flat Plate," *The Aeronautical Journal*, Vol. 85, 1981, Paper 845, pp. 34-35.

[6]Töpfer, C., "Bemerkungen zu dem Aufsatz von H. Blasius," *Zeitschr. f. Math. a. Phys.*, Vol. 60, 1912, pp. 397-398.

[7]Ben-Dor, G., Rakib, Z., and Igra, O., "The Boundary Layer of a Compressible Singly Ionized Frozen Flow over a Flate Plate—A New Method of Solution," Collection of Papers of the 25th Israel Annual Conference on Aviation and Astronautics, 1983.

Calculation of Three-Dimensional Instability of a Blasius Boundary Layer

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Introduction

TRANSITION in a boundary layer below a freestream of low turbulence level and small spatial variations consists of the following stages: 1) the appearance of two-dimensional Tollmien-Schlichting waves; 2) with the increase of the Reynolds number as flow goes downstream, three-dimensional waves appear—the subject of the present Note; 3) the onset of nonlinear effects as the wave amplitude grows; and 4) the onset of randomness beginning as modulation of the higher harmonics in the concentrated shear layers.

Stage 1 can be well described by linear theory for two-dimensional infinitesimal disturbances. The third stage is distinguished by the onset and increase of the nonlinear effects. Perhaps the problems involved in this state are the most important and complicated in the transition process.

Wortmann[1] proposed a model for the disturbance structure in the second, incipiently three-dimensional stage, based on his own measurements. The disturbance was proposed to consist of longitudinal counterrotating sheets of vorticity, inclined downstream and overlapping each other like roof shingles. Complete three-dimensional solutions, even of linearize equations, would be very time-consuming. In this Note we show that Squire's theorem[2] relating two and three-dimensional disturbances, together with the fact that (longitudinal) wave number α appears in the Orr-Sommerfeld linearized stability equation only in the group αR (where R is the Reynolds number), can be used to relate known two-dimensional eigenfunctions to a small three-dimensional disturbance with the same αR, but with a *resultant* wave number $\sqrt{(\alpha^2+\beta^2)}$ equal to the two-dimensional α. We consider only neutral disturbances for simplicity, but this is not a necessary restriction.

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Small-Disturbance Theory

Following Benney[3] we assume that the small three-dimensional disturbance has the form

$$u=\hat{u}(y)e^{i\alpha(x-ct)}\cos\beta z \tag{1a}$$

$$v=\hat{v}(y)e^{i\alpha(x-ct)}\cos\beta z \tag{1b}$$

$$w=\hat{w}(y)e^{i\alpha(x-ct)}\sin\beta z \tag{1c}$$

$$p=\hat{p}(y)e^{i\alpha(x-ct)}\cos\beta z \tag{1d}$$

where the y-dependent amplitudes indicated by a caret above the quantity are the "disturbance wave functions" and the $\sin\beta z$ factor in w is necessary to satisfy the continuity equation. Substituting into the Navier-Stokes equations and linearizing, we obtain a system of equations for the disturbance wave functions, on a two-dimensional mean flow $U(y)$, as

$$\left(\frac{d^2}{dy^2}-\gamma^2\right)^2\hat{v}=i\alpha R\left\{(U-c)\left(\frac{d^2}{dy^2}-\gamma^2\right)\hat{v}-\frac{d^2U}{dy^2}\hat{v}\right\} \tag{2}$$

$$\left\{\frac{d^2}{dy^2}-\gamma^2-i\alpha R(U-c)\right\}\hat{\omega}_2=-\beta R\frac{dU}{dy}\hat{v} \tag{3}$$

$$\gamma^2\hat{u}=i\alpha\frac{d\hat{v}}{dy}-\beta\hat{\omega}_2 \tag{4a}$$

$$\gamma^2\hat{w}=i\alpha\hat{\omega}_2-\beta\frac{d\hat{v}}{dy} \tag{4b}$$

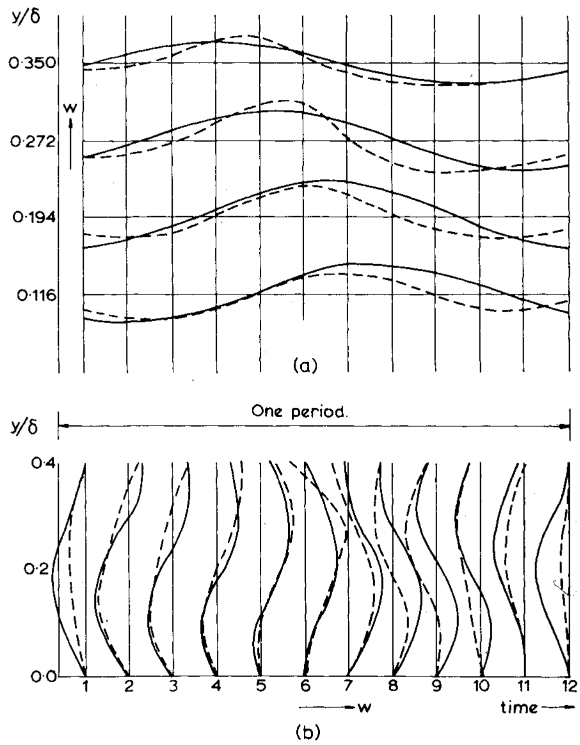

Fig. 1 Variation of velocity w with time over one period at a fixed x position for different heights: 1) $w(t)$ at different values of y/δ; b) $w(y)$ at 30 deg phase intervals [—, present calculations; ———, experiment (Ref. 1)].